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H. BORG

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THE USE OF PERMEABILITY, CONDUCTIVITY, CONDUCTANCE
AND RESISTANCE IN THE DESCRIPTION OF
WATER MOVEMENT IN SOILS AND PLANTS
TECHNICAL REPORT NO 71

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ABSTRACT

Permeability, conductivity, conductance and resistance are important parameters in the description of water movement in soils and plants. These terms are related, but not synonymous. Permeability is only a function of the flowpath geometry of the medium, while conductivity also depends on the fluid involved. Conductance, in addition, takes the size of the medium into account. Resistance is simply the inverse of conductance. By applying the definition of resistance, it is shown that Darcy's law and Ohm's law are equivalent.

Conductance and resistance relate volume flow rate to a potential difference, but to relate volume flux (volume flow rate per unit area) to a potential difference an area specific conductance or an area specific resistance must be used. Permeability and conductivity can be used in both cases.

Conductances and resistances are also employed in the description of vapour movement where they relate mass flow rate or mass flux to a concentration difference. Their dimensions are therefore different to those of the analogous terms in the description of water movement. However, they can be made identical by introducing proper conversion factors. By doing so, it is shown that Darcy's law and Fick's law are related.
INTRODUCTION

Water moves in response to a difference in water potential between two points in or across a medium. For a given potential difference the magnitude of the flow depends on the ease with which water can move in the medium. This ease of movement can be represented by the permeability, conductivity, conductance or resistance of the medium.

Although these terms are related, they are not synonymous and therefore cannot be used interchangeably. Nevertheless, it is frequently done and thus causes confusion in the analysis of experimental data. For instance, Ponsana (1975) states that axial and radial root resistance cannot be compared directly because they have different dimensions. However, what he defines as axial root resistance is actually axial root resistance per unit length of root, and his radial root resistance is actually radial root resistance multiplied by root length. Axial and radial root resistance do have the same units, but not the parameters he defines as such. A summary of Ponsana’s work is presented by Greacen et al. (1976).

As another example, water movement across plant membranes can be described by equation (10) given below, where $L_A$ is the area specific conductance. However, Newman (1976) calls $L_A$ permeability, Wendler and Zimmermann (1982) and Passioua (1984) refer to it as conductivity, and Shalhevet et al. (1976) as conductance. Also, in the older soil physics literature, permeability is often used as a synonym for conductivity (Carman 1937; Childs and Collis-George 1950).

This paper reviews the definitions of permeability, conductivity, conductance and resistance. All four terms are essentially proportionality factors which relate volume flow rate to a water potential difference. Their dimensions therefore depend on the dimensions assigned to water potential. Volume flow rate has dimensions of length-cubed per
time \( [L^3/T] \), and in this article water potential is expressed as energy per unit weight, which gives it dimensions of length \([L]\). From this follow the dimensions given to permeability, conductivity, conductance and resistance below.

**Definition of Terms**

The ease with which a fluid moves through a medium is determined by three factors, namely:

(1) the flowpath, i.e. the size and shape of the flow channels occupied by the fluid under consideration;

(2) the fluid properties, i.e. the density and viscosity of the fluid;

(3) the size of the medium, i.e. the cross-sectional area and thickness (length) of the medium.

PERMEABILITY is the facility with which a medium transmits fluids. It depends only on the flowpath geometry, but not on the fluid or the size of the medium (Hubbard 1940). It has dimensions of length-squared \([L^2]\) and is usually represented by the letter \(k\). Permeability is widely used in the petroleum industry where oil, gas and water are often present in a multiphase flow system. The use of fluid-independent parameters simplifies the description of such a flow system.

CONDUCTIVITY is the facility with which a medium transmits a particular fluid. It is a function of the flowpath geometry and fluid properties, but not of the size of the medium (Hubbard 1940). Its dimensions are length per time \([L/T]\) and it is usually represented by the letter \(K\). Conductivity is related to permeability by the equation

\[
K = \frac{k \rho g}{\mu}
\]  

(1)
where \( \rho = \) fluid density [M/L\(^3\)], \( g = \) acceleration due to gravity [L/T\(^2\)], and \( \mu = \) fluid viscosity [FT/L\(^2\) = M/L/T].

CONDUCTANCE is the facility with which a medium of specified size transmits a particular fluid. It depends on the flowpath geometry, fluid properties, and the dimensions of the medium (Weast 1978). It is generally represented by the letter C or L. The latter will be used in this article. Conductance is related to the conductivity and the size of the medium as

\[
L = \frac{KA}{l}
\]

(2)

where \( A = \) cross-sectional area of the medium [L\(^2\)], and \( l = \) thickness (length) of the medium [L]. \( K \) has the dimensions [L/T] so that conductance has dimensions of [L\(^2\)/T]. Note that the dependence of conductance on flowpath geometry and fluid properties enters through the presence of \( K \) in equation (2).

RESISTANCE is the reciprocal of conductance. It is abbreviated as \( R \) and related to the conductivity and the size of the medium by the equation

\[
R = \frac{9}{KA}
\]

(3)

which is the inverse of (2). Therefore, resistance has dimensions of [T/L\(^2\)].

RESISTIVITY is the inverse of conductivity, which gives it dimensions of [T/L]. This parameter is hardly ever used in the description of water movement.

Conductivity, conductance, resistance and resistivity sometimes receive the prefix 'hydraulic' (e.g. hydraulic resistance) to specify that water is the fluid under consideration.
Equations for the Description of Water Movement in Soils and Plants

To describe water movement in soils, Darcy (1856) experimentally derived the relationship

\[ Q = K A \frac{\Delta \psi}{l} \]  \hspace{1cm} (4)

where \( Q \) = volume flow rate \([L^3/T]\), and \( \Delta \psi \) = water potential difference across a medium of length \( l \). Darcy's work is reviewed in detail by Hubbard (1940). Substituting (2) into (4) yields

\[ Q = L \Delta \psi \]  \hspace{1cm} (5)

and substituting (3) into (4) gives

\[ Q = \frac{\Delta \psi}{P} \]  \hspace{1cm} (6)

which is a hydraulic analogue of Ohm's law. Equations (4), (5) and (6) are equivalent and equally applicable to the description of water movement in soils and plants. However, Darcy's law is mostly applied to water flow in soils, while the Ohm's law analogue is mostly applied to water movement through plants. Equation (5) is widely used to describe water flow across plant membranes.

It is often preferable to discuss water movement in terms of volume flux rather than volume flow rate. Volume flux \( q \) is the volume flow rate per unit area. It is computed as \( q = Q/A \), which gives it dimensions of \([L^3/T/L^2]\) (Hillel 1980). To write Darcy's law in terms of volume flux, both sides of (4) are divided by \( A \) which gives

\[ q = K \frac{\Delta \psi}{l} \]  \hspace{1cm} (7)

Similarly, dividing equation (5) by \( A \) yields

\[ q = \frac{L}{A} \Delta \psi \]  \hspace{1cm} (8)
An inspection of equation (2) shows that \( L/A = K/L \). One may define this term as area specific conductance \( L_A \), where

\[
L_A = \frac{K}{L}
\]

whose dimensions are \([1/T]\). The subscript \( A \) is introduced to indicate that the cross-sectional area of the medium has been cancelled out. The coupled transport of water and solutes across plant membranes is generally described by an equation of form (8) as

\[
q = L_A (\Delta P + \sigma \Delta \pi)
\]

where \( \Delta P = \) hydraulic pressure potential difference \([L]\) across the medium, \( \sigma = \) dimensionless solute reflection coefficient, and \( \Delta \pi = \) osmotic potential difference \([L]\) across the membrane.

Ohm's law in terms of flux becomes

\[
q = \frac{\Delta \psi}{R_A}
\]

A comparison with equation (3) shows that \( RA = \ell/K \). In analogy to \( L_A \) one may define area specific resistance \( R_A \) as

\[
R_A = \frac{\ell}{K}
\]

which has dimensions of \([T]\). The subscript \( A \) again indicates that the cross-sectional area of the medium has been eliminated.

A widespread application of equation (6) is the description of transpiration as

\[
Q = \frac{\Delta \psi}{R} = \frac{\psi_{soil} - \psi_{leaf}}{R_{soil} + R_{plant}}
\]

where the superscripts denote soil and leaf water potential, and soil and plant resistance, respectively.

In the transpiration process, water moves sequentially through the soil, root system, stem and leaves. Each of these media
has a different cross-sectional area. Often a transpiration flux $T$ is used to represent the volume flow rate per unit leaf area, per unit projected canopy area, or per unit ground area. It is computed as $T = Q/a$, where $a$ represents either of the aforementioned areas. Substituting this relationship into equation (13) produces an expression of form (11) where

$$T = \frac{\Delta \psi}{\text{Ra}} \quad (14)$$

It follows from equation (3) that

$$\text{Ra} = \frac{\frac{\mu}{a}}{K_A} \quad (15)$$

The variables $a$ and $A$ represent different areas, but both have the dimensions $[L^2]$. The product $\text{Ra}$ therefore has the same dimensions as area specific resistance, namely $[T]$. One may regard it as another form of area specific resistance and define $R_a = \text{Ra}$, where the subscript $a$ indicates that resistance has been multiplied by some arbitrary area $a$. Observe that area specific resistance and area specific conductance relate volume flux to a potential difference, while resistance and conductance relate volume flow rate to a potential difference.

Equations for the Description of Vapour Movement in Soils and Plants

The diffusion of vapour is described by the equation

$$F = D \frac{A_c}{\ell} \quad (16)$$

where $F =$ mass flow rate $[M/T]$, $D =$ diffusion coefficient $[L^2/T]$, and $c =$ vapour concentration $[M/L^3]$. However, the flux version of equation (16) is used almost exclusively to describe vapour movement as

$$f = D \frac{A_c}{\ell} \quad (17)$$
where \( f = \frac{F}{A} \) = mass flux \([M/L^2T]\). This relationship is known as Fick's first law of diffusion. It is common practice (Monteith 1973; Campbell 1977) to define conductance to vapour movement \( g \) as

\[
g = \frac{D}{L}
\]

and resistance to vapour movement \( r \) as

\[
r = \frac{L}{D}.
\]

The dimensions of \( g \) are \([L/T]\), and \( r \) has dimensions of \([T/L]\). Substituting (18) into (17) yields

\[
f = g \Delta c
\]

and substitution of (19) into (17) gives

\[
f = \frac{\Delta c}{r}.
\]

Equations (20) and (21) are widely used to describe evaporation from plant leaves (Slatyer 1967; Nobel 1983) where \( g \) is then referred to as leaf or stomatal conductance, and \( r \) as leaf or stomatal resistance.

Note that the derivations of \( g \) and \( r \) are based on a flux equation. Therefore, \( g \) is in fact the area specific conductance, and \( r \) the area specific resistance to vapour movement. Their dimensions, namely \([L/T]\) and \([T/L]\), are different to those of area specific conductance and area specific resistance to water movement, which are \([L^2/T]\) and \([T/L^2]\), because \( g \) and \( r \) relate mass flux to a concentration difference, while \( L_A \) and \( R_A \) relate volume flux to a potential difference.
The Relationship between $L_A$ and $g_A$, and $R_A$ and $r_A$

Consider the differential form of Fick's law

$$f = D \frac{dc}{d\ell}.$$  \hspace{1cm} (22)

Vapour concentration can be given as

$$c = c_0 h.$$ \hspace{1cm} (23)

where $c_0$ = saturation vapour concentration [M/L$^3$], and $h$ = relative humidity [dimensionless]. Campbell (1977) shows that relative humidity and water potential are related as

$$h = \exp\left(\frac{m \psi}{R T}\right).$$ \hspace{1cm} (24)

where $m$ = molecular weight of water [dimensionless], $R$ = gas constant [J/(mol/K)], and $T$ = absolute temperature [K]. Using (23) one can write

$$\frac{dc}{d\ell} = c_0 \frac{dh}{d\ell}. \hspace{1cm} (25)$$

Applying the chain rule to (25) gives

$$\frac{dh}{d\ell} = \frac{dh}{d\psi} \frac{d\psi}{d\ell}. \hspace{1cm} (26)$$

Combining (24) and (25) then yields

$$\frac{dh}{d\psi} = \frac{m \psi}{RT} \exp\left(\frac{m \psi}{RT}\right) = \frac{m \psi}{RT} h. \hspace{1cm} (27)$$

Substituting (25), (26) and (27) into (22) leads to

$$f = D \frac{c_0 \frac{m \psi}{RT} \psi}{d\ell}. \hspace{1cm} (28)$$

Mass flux and volume flux are related by

$$q = f.$$ \hspace{1cm} (29)

Substituting (29) into (28) and integrating produces

$$q = D \frac{c_0 \frac{m \psi}{RT} \Delta\psi}{\rho RT \ell}.$$ \hspace{1cm} (30)
Comparing (30) to (7) then shows that

\[ K = D \frac{cm\gamma}{\rho R \tau} \]  

(31)

from which follows that

\[ L_A = g \frac{cm\gamma}{\lambda R \tau} \]  

(32)

and

\[ R_A = \frac{\Gamma}{\rho R \tau} \frac{\lambda R \tau}{cm\gamma} \]  

(33)

In (28) and (30) to (33), \( c \) and \( \tau \) represent an average value for the medium through which the vapour passes.

Acknowledgements

Many thanks to Fiona Carter and Karen Lemnell for their help in preparing this manuscript.
## Glossary of Terms

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>cross-sectional area of the medium</td>
<td>( [L^2] )</td>
</tr>
<tr>
<td>D</td>
<td>diffusion coefficient</td>
<td>( [L^2/T] )</td>
</tr>
<tr>
<td>F</td>
<td>mass flow rate</td>
<td>( [M/T] )</td>
</tr>
<tr>
<td>K</td>
<td>conductivity</td>
<td>( [L^2/T] )</td>
</tr>
<tr>
<td>L</td>
<td>conductance</td>
<td>( [L^2/T] )</td>
</tr>
<tr>
<td>( L_A )</td>
<td>area specific conductance</td>
<td>( [1/T] )</td>
</tr>
<tr>
<td>P</td>
<td>hydraulic pressure potential</td>
<td>( [L] )</td>
</tr>
<tr>
<td>Q</td>
<td>volume flow rate</td>
<td>( [L^3/T] )</td>
</tr>
<tr>
<td>R</td>
<td>gas constant</td>
<td>( [FL/M/K] )</td>
</tr>
<tr>
<td>R</td>
<td>resistance</td>
<td>( [T/L^2] )</td>
</tr>
<tr>
<td>( R_A, R_a )</td>
<td>area specific resistance</td>
<td>( [T] )</td>
</tr>
<tr>
<td>T</td>
<td>transpiration flux</td>
<td>( [L^3/T/L^2] )</td>
</tr>
<tr>
<td>c</td>
<td>vapour concentration</td>
<td>( [M/L^3] )</td>
</tr>
<tr>
<td>( c_o )</td>
<td>saturation vapour concentration</td>
<td>( [M/L^3] )</td>
</tr>
<tr>
<td>f</td>
<td>mass flux</td>
<td>( [M/T/L^2] )</td>
</tr>
<tr>
<td>g</td>
<td>gravitational acceleration</td>
<td>( [M/T^2] )</td>
</tr>
<tr>
<td>g</td>
<td>area specific (vapour) conductance</td>
<td>( [L/T] )</td>
</tr>
<tr>
<td>h</td>
<td>relative humidity</td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>permeability</td>
<td>( [L^2] )</td>
</tr>
<tr>
<td>( \ell )</td>
<td>thickness or length of the medium</td>
<td>( [L] )</td>
</tr>
<tr>
<td>m</td>
<td>molecular weight</td>
<td></td>
</tr>
<tr>
<td>q</td>
<td>volume flux</td>
<td>( [L^3/T/L^2] )</td>
</tr>
<tr>
<td>r</td>
<td>area specific (vapour) resistance</td>
<td>( [T/L] )</td>
</tr>
<tr>
<td>( \psi )</td>
<td>water potential</td>
<td>( [L] )</td>
</tr>
<tr>
<td>( \tau )</td>
<td>absolute temperature</td>
<td>( [K] )</td>
</tr>
<tr>
<td>( \mu )</td>
<td>fluid viscosity</td>
<td>( [FT/L^2] )</td>
</tr>
<tr>
<td>( \pi )</td>
<td>osmotic potential</td>
<td>( [L] )</td>
</tr>
<tr>
<td>( \rho )</td>
<td>fluid density</td>
<td>( [M/L^3] )</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>solute reflection coefficient</td>
<td></td>
</tr>
</tbody>
</table>
Basic dimensions

[L] = arbitrary dimensions of length
[M] = arbitrary dimensions of mass
[T] = arbitrary dimensions of time
[F] = [ML/T^2] = arbitrary dimensions of force
[K] = degrees Kelvin
REFERENCES


